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Development of a Mathematical Model for Estimating Rock Thermal Conductivity Using Analogy between Electricity Transmission and Heat Transfer

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Introduction

Thermal conductivity of a porous medium depends on several factors such as matrix properties (for example mineralogical composition, grain shape, grain size and cementation), pore space properties (such as porosity, porosity type, geometrical configuration of pores, distribution of the pores, pore filling fluids and their saturations), temperature range and the exerted stress on the rock [1-6]. Also, prediction of thermal conductivity of porous media is associated with high level of uncertainty due to their complicated structure. Moreover, numerous experimental methods, empirical correlations and theoretical models have been reported in the literature to predict thermal conductivity of a saturated porous medium. But most of them are limited to the porous medium which is saturated with single phase [7].

In this paper, thermal conductivity of six carbonate rock samples from an Iranian reservoir was measured at vacuum condition (λ_{vac}). In addition, thermal conductivity of four plugs of these six samples was determined at fully water saturated (λ_{ew}) and partially water saturated (λ_{ep}) conditions. Also, based on the analogy between electricity transmission and heat transfer through porous media, a new model was presented to predict rock thermal conductivity at different water saturations.

Development of a new model for heat transfer through a partially water saturated rock

Analogy between electricity transmission and heat transfer can be used to establish a model to predict thermal conductivity. Archie's second equation, i.e. Equation 1, is used to investigate this analogy. This equation extends relationship between electrical resistivity index (I_R) and water saturation (S_w):

$$I_R = \frac{\sigma_o}{\sigma_t} = S_w^{-n} \tag{1}$$

where, σ_o, σ_t and n are electrical conductivities of fully water saturated rock, electrical conductivity of partially water saturated rock and saturation exponent respectively.

If Equation 1 is written for heat transfer through a partially water saturated rock, Equation 2 is obtained:

$$T_R = \frac{\lambda_{ew}}{\lambda_{ep}} = S_w^{-t} \qquad (2)$$

where, fraction of $\lambda_{ew}/\lambda_{ep}$ and t are called as thermal resistivity index (T_R) and thermal saturation exponent respectively.

A closer look at the Equation 1 reveals that the electrical conductivity of partially saturated rock (denominator of Equation 1) is just a function of the amount of water in the pore space. Because air and rock matrix are very poor electrical conductors. The numerator of Equation 1 includes electrical conductivity of fully water saturated rock. Therefore, resistivity index, as a ratio of mentioned numerator and denominator, would be a function of water saturation. Now, if we want to write an equation similar to Equation 1 based on the analogy between electricity transmission and heat transfer, Equation 2 is obtained. But by making a comparison between Equation 1 and Equation 2, a problem is revealed. Despite the Equation 1, at the denominator of Equation 2, beside water, air and rock matrix conduct heat as well. Also, at the numerator of Equation 2, water and rock matrix transfer heat. Thus, in general, Equation 2 is not just a function of water saturation. Hence, this equation cannot be used same as Equation 1.

To resolve this problem, a new model is introduced in which the numerator and denominator are functions of thermal conductivity of resident fluids solely. Therefore, the numerator is defined as λ_{ew}-λ_{vac}, in which the effect of rock matrix has been excluded by considering λ_{vac}. λ_{vac} is thermal conductivity of the rock sample at vacuum condition. In other words, this parameter implies thermal conductivity of the rock sample when it contains no fluid. Also, in the new model, the denominator is expressed as λ_{ep}-λ_{vac} which implies that denominator is just function of thermal conductivity of saturating fluids. In this way, it is expected that the quotient of defined numerator and denominator (f(λ)=$\frac{\lambda_{ew}-\lambda_{vac}}{\lambda_{ep}-\lambda_{vac}}$) is a function of water saturation.

Experimental procedure

Divided bar steady-state technique was applied for measuring thermal conductivity of six carbonat plug samples at vacuum condition, fully water saturated and partially water saturated conditions (four different water saturations).

Results and Discussion

In Figure 1, thermal conductivity of six plugs at vacuum condition versus porosity is depicted. It is evident that λ_{vac} increases as porosity decreases.

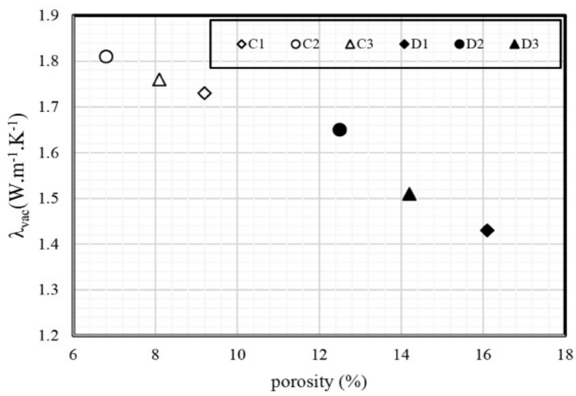

Figure 1: Thermal conductivity at vacuum condition versus porosity for six plugs.

Thermal conductivity of four plug samples, *C1*, *C2*, *D1* and *D2*, was measured at five different degrees of water saturation. One of these saturations was 100% water saturation. The second phase in these experiments was air. Figure 2 shows thermal conductivity of partially saturated plugs (λ_{ep}) versus water saturation. It is obvious that λ_{ep} increases with an increase in water saturation or decreasing air content. This is due to the higher value of water thermal conductivity with respect to air thermal conductivity. Mineralogical compositions of plug samples *D2* and *D1* are the same. Therefore, higher value of λ_{ep} of plug *D2* is related to its lower porosity. In addition, λ_{ep} of plugs *C1* and *D2* are close to each other. Although the plug *C1* contains calcite with lower thermal conductivity than dolomite, plug *C1* has lower porosity than plug *D2*. These two opposite effects lead to closeness of λ_{ep} of plugs *C1* and *D2*. Also, Figure 2 illustrates that λ_{ep} of plug *C1* is obviously greater than plug *D1* that is related to significant difference between their porosities. Plug *C2* has the highest λ_{ep} because it has the lowest value of porosity.

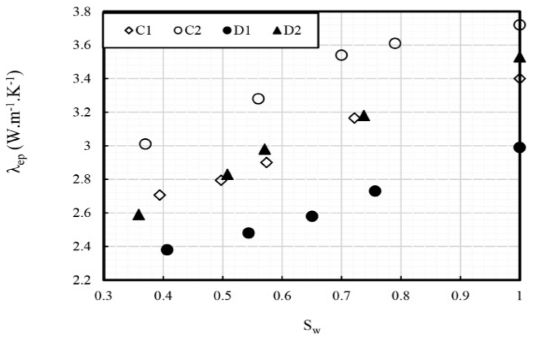

Figure 2: Thermal conductivity versus water saturation.

f(λ) was plotted versus water saturation for all plugs in Figure 3 to investigate applicability of the new model. Furthermore, power equations were fitted to the data of Figure 3 to examine similarity between *f(λ)* in the new model and I_R in Archie's equation.

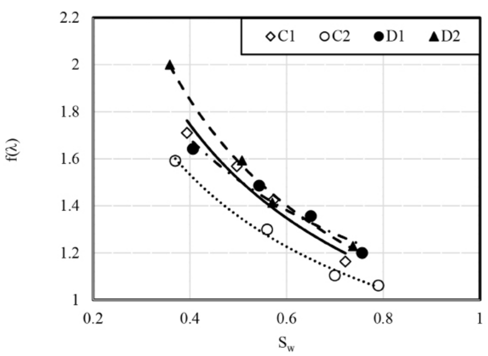

Figure 3: *f(λ)* versus water saturation for all plugs.

Table 1 presents fitted equations and their corresponding determination coefficients *(R^2)*. High value of R^2 of fitted equations confirms similarity between the proposed model of λ_{ep} and second Archie's equation. Also, this table reveals that saturation exponent in the new model changes from 0.49 to 0.69 which is very different from typical value of saturation exponent *(n=2)* in Archie's second equation. Moreover, this implies that water saturation affects the heat transfer and electricity transmission in two different ways. Last row of Table 1 shows power equation that includes thermal conductivity data of three plugs.

Table 1: Fitted equations and their corresponding determination coefficients of data in Figure 3.

Plug No.	Fitted equation	R^2
C1	$f(\lambda) = 0.97S_w^{-0.64}$	0.96
C2	$f(\lambda) = 0.95S_w^{-0.55}$	0.99
D1	$f(\lambda) = 1.08S_w^{-0.49}$	0.96
D2	$f(\lambda) = 0.99S_w^{-0.69}$	0.99
All plugs	$f(\lambda) = 0.99S_w^{-0.60}$	0.93

CONCLUSIONS

In this paper, thermal conductivity of six carbonate plugs was measured at vacuum condition, fully water saturated and partially saturated conditions. Results showed that thermal conductivity at vacuum condition increases with a decrease in porosity. Also, it was observed that thermal conductivity increases with an increase in water saturation (S_w). Furthermore, a new model was suggested to predict rock thermal conductivity based on the analogy between heat transfer and electrical flow. Finally, the results revealed that function *f(λ)*, in the new model, is a power function of S_w similar to the dependency of resistivity index on S_w in Archie's equation. But the exponent of S_w in the new model changes from 0.49 to 0.69 which is different from the typical value of saturation exponent in Archie's equation *(n=2)*.

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